FINESTRUCTURE OF TRIODE CHARACTERISTICS

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Summary

The detailed structure of diode-, triode- and tetrode-characteristics is experimentally investigated by measuring the amplitudes of the different harmonics (up to the tenth) in the anodecurrent, as a function of the grid bias potential, when a sinusoidal e.m.f. of constant amplitude is applied to the grid. These curves show a great number of maxima and minima. The amplitude of every next harmonic, considered as a function of the grid biassing potential has, to a pronounced degree of accuracy, the form of the differential of the former one. It is thus possible to determine the successive differentials of the characteristic in a wide domain. However, in some small domains this appears not to be the case. The latter anomalies in the higher differentials of the characteristic are most likely due to critical potentials in the secondary electron emission from the electrodes.

A new analytical development of the characteristic in a series of Tchebycheff polynomials is given which has some advantages over the development in a Taylorseries. Whereas the coefficients of the Tchebycheff series follow immediately from the measurements with a finite amplitude on the grid, the coefficients of the Taylor series can only be derived from the limit, towards which these measurements converge for an infinitely small gridswing.

§ 1. Although in many elementary textbooks triode characteristics are often idealised to a linear relation between anode current i_a, grid potential V_g and anode potential V_a, actual measurements and many oscillation phenomena are decidedly contradictory to this simple point of view. The fact already e.g. that a triode, connected to an oscillatory circuit and with retroaction, produces oscillations of a finite and constant amplitude, is a proof of the non-linearity of the functions, expressing the relations between i_a, i_g, V_a and V_g.

The aim of the present paper is to investigate the detailed and microscopic structure of triode characteristics in that range of the variables V_g, V_a and i_a, which is usually employed when the triode is used as an amplifier or oscillator. A detailed knowledge of the non-linearity of triode characteristics provides an insight into the inherent distortion, produced by a triode used as an amplifier, and further into such phenomena as „crossmodulation'' and the limiting amplitudes, occurring when an external sinusoidal e.m.f. is impressed upon a triode receiver, provided with critical retroaction, etc.

§ 2. In the present method of investigating the finestructure of the $i_a — V_g$ characteristics (see fig. 1) a carefully filtered and therefore pure sinusoidal e.m.f. was impressed on the grid of a triode, in the anode of which a resistance R_a was inserted, small compared with the internal differential anode resistance of the triode (or tetrode or penthode) under investigation.

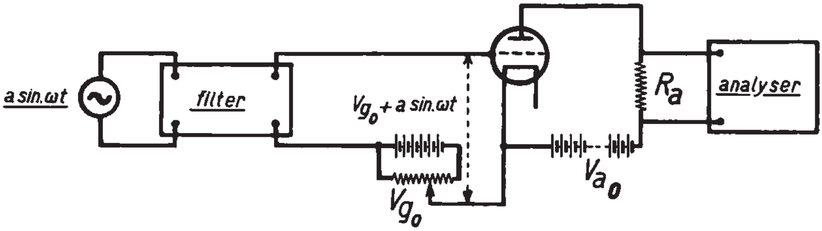

Fig. 1

Thus the anode potential variations are so small, that they practically have no influence on the anode current, as was confirmed experimentally.

If the characteristic function $i_a = f(V_g)$ were linear, the potential difference, developing over R_a would also be purely sinusoidal. Due, however, to the actual non-linear relation between the variables i_a and V_g, the potential, developing over R_a, contains several measurable (up to the tenths) harmonics.

The set-up contained a triode oscillator ($\lambda = 1700$ m). The oscillations were filtered by a 5 section lowpass filter of such a kind, that the second harmonic of this input was smaller than 1/10000 of the amplitude of the fundamental or first harmonic. The amplitudes of the third and higher harmonics were still considerably smaller than that of the second harmonic.

The pure sinusoidal e.m.f. thus obtained acted on the grid of the tube under investigation, in series with an adjustable constant voltage V_{g_0}. The internal impedance of the filter circuit was sufficiently small to let the grid potential remain sinusoidal, even when some grid current was flowing. In the anode circuit of the tube a wire wound resistance R_a was inserted of 50 Ω for a triode and 1000 Ω for a tetrode or penthode. The different harmonics of the alternating output voltage $(R_a i_a)$ were filtered in an analyser by two oscillatory circuits, both of which were tuned to the harmonic under investigation.

The amplitude of the harmonic thus filtered was measured with the aid of a superheterodyne system. The intermediate frequency stages of this superheterodyne were provided with an attenuator. This intermediate frequency stage was followed by a diode rectifier. The change in the anode current of this rectifier was amplified once by a triode, acting as a d.c. amplifier. The change of the anode current of this triode was, with the aid of the attenuator, constantly fixed at 4 mA. From a calibration of the apparatus and from the setting of the attenuator the amplitude of the harmonic under investigation could thus be simply determined. Of course the whole apparatus had to be very carefully shielded.

In order to verify the correct functioning of the apparatus, e.g. when connected for the investigation of the characteristic of a diode, this diode was replaced by a resistance wire, having about the same resistance as the diode and under these circumstances not the slightest trace of any harmonic could be found in the analyser. This experiment was considered as a proof of the reliability of the results obtained.

A complete series of measurements ran as follows.

After the tube under investigation was put in the socket, a constant anode potential V_{a_0} was applied to it. This potential was kept constant thoughout the complete series of measurements. First, by varying the negative grid potential V_{g_0} step by step, the corresponding static characteristic $i_{a_0} = f(V_{g_0})$ was determined. Thereupon the sinusoidal e.m.f., (varying in different experiments between 0.05 and 10 Volt R.M.S.) was applied to the grid. The analyser was tuned to this fundamental frequency (first harmonic) and the amplitude of this first harmonic in the anode current was measured as a function of the negative grid bias potential V_{g_0}. Thereupon the analyser was tuned to the second harmonic and, for the same input amplitude, the amplitude of the second harmonic in the anode current was in its turn measured as a function of the grid bias potential V_{g_0}. Similarly for the third harmonic, etc. etc. In this way our apparatus enabled us to take reliable measurements up to the tenth harmonic.

§ 3. In fig. 2—6 are shown the results of a series of measurements, made on a Philips E 452 T tetrode. It contains an indirectly heated cathode ($V_f = 4$ Volt, $i_f = 1$ Amp.), the anode potential was $V_{a_0} = 200$ Volt, the potential of the screen grid $V'_{g_0} = 100$ Volt. In this series of measurements the sinusoidal e.m.f., applied to the grid, was 1 Volt R.M.S. In these figures i_{a_0} represents the static characteristic, i_1 the R.M.S. of the first harmonic of the anode current, i_2 the R.M.S. of the second harmonic, etc. etc.

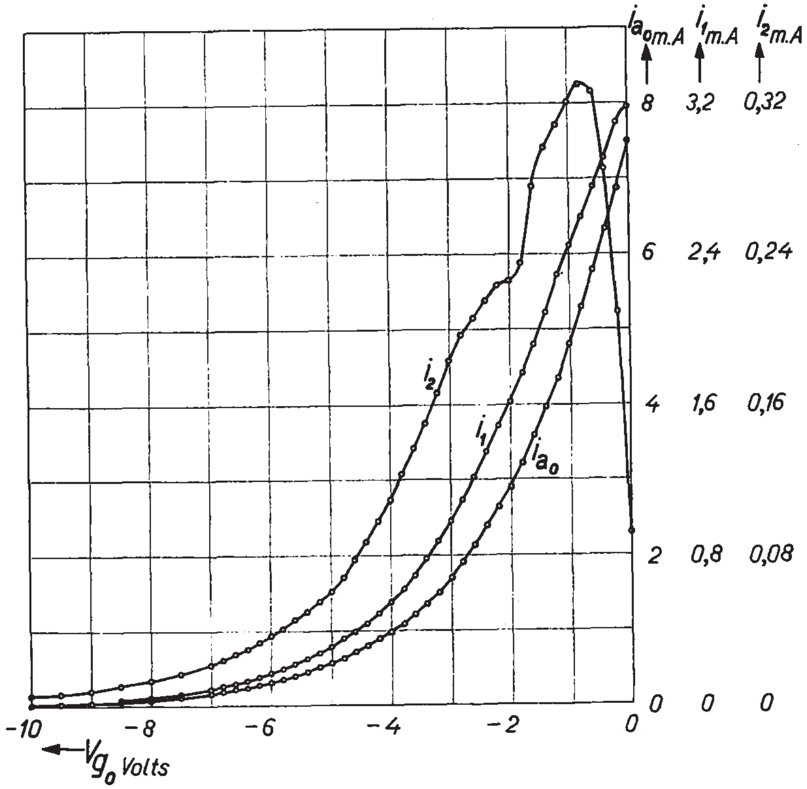

Fig. 2

Similarly fig. 7—10 represent the measurements on a Philips directly heated tetrode A 442 ($V_f = 4$ Volt, $i_f = 0.06$ Amp.,

$V_{a_0} = 200$ Volt, $V'_{g_0} = 100$ Volt, input voltage again 1 Volt R.M.S.).

In fig. 11 we give i_{a_0} and i_2 of a Philips electrometertriode type 4060 ($V_f = 0.6$ V, $i_f = 1$ Amp) with the anode and control electrode connected so that the tube was used as a diode. Similarly fig. 12 represents the measurements made on an indirectly heated cylindrical diode ($V_f = 4$ Volt, $i_f = 1$ Amp) anode diameter 4 mm). In the case of a diode or a triode used as a diode, the amplitudes of the harmonics are represented as a function of the mean anode potential V_{a_0}. The alternating e.m.f. applied in the anode circuit of the diodes was again 1 Volt R.M.S.

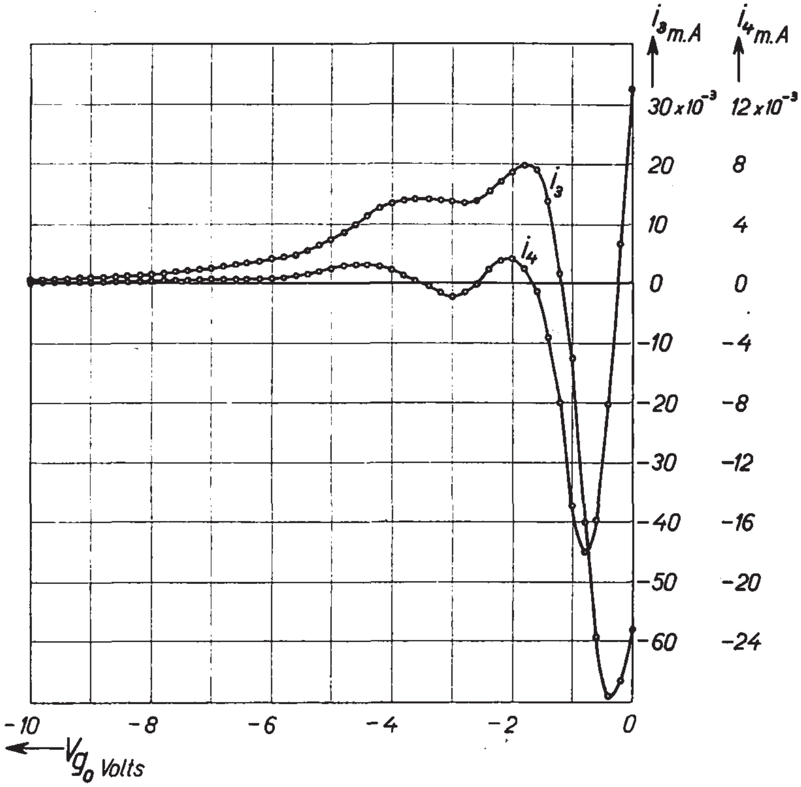

Fig. 3

It may here be noted that when measurements were repeated at intervals of some weeks, the results could be accurately reproduced even for the higher harmonics; also different triodes of the same type yielded remarkably similar results.

§ 4. From an examination of figures 2—6 and 7—10, which show a considerable number of maxima and minima, it is at once clear, that every curve of the same set has, to a pronounced degree of accuracy, the form of the differential of the one preceding it. In order to demonstrate this clearly and although amplitudes, not phases, were measured, our curves were plotted partly above and partly below the axis of abscissae, so as to show smooth functions. In no case was there any doubt as to whether a point had to be located above or below the axis of abscissae.

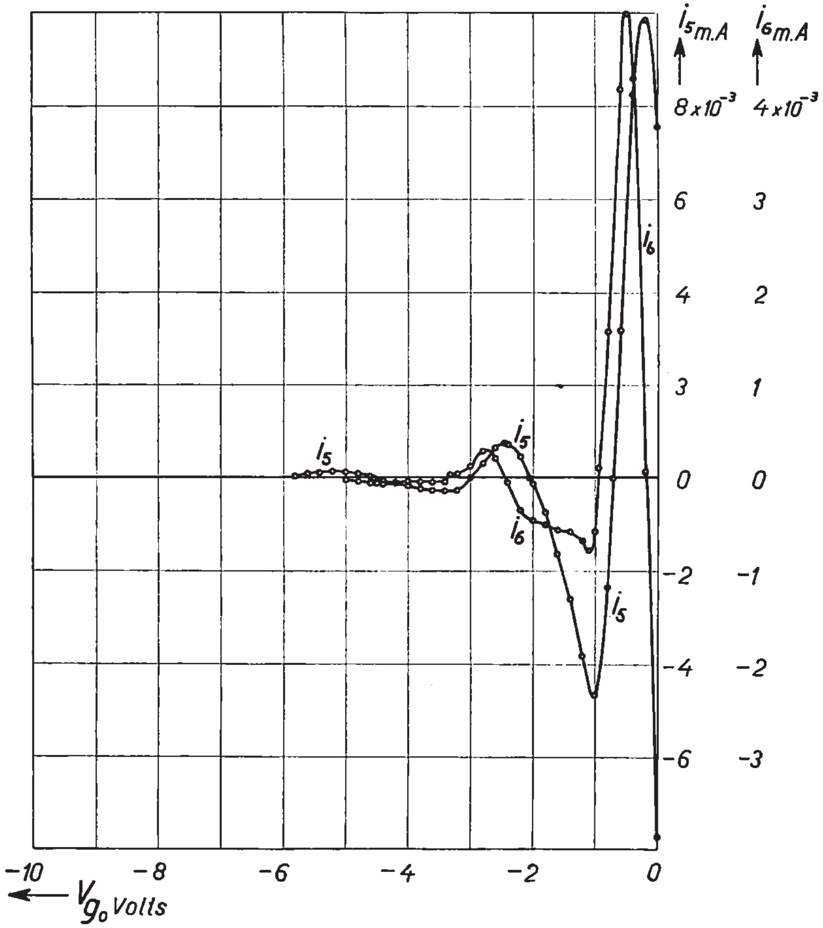

Fig. 4

That it is indeed likely that every curve has the form of the differential of the one preceding it, follows from the following reasoning.

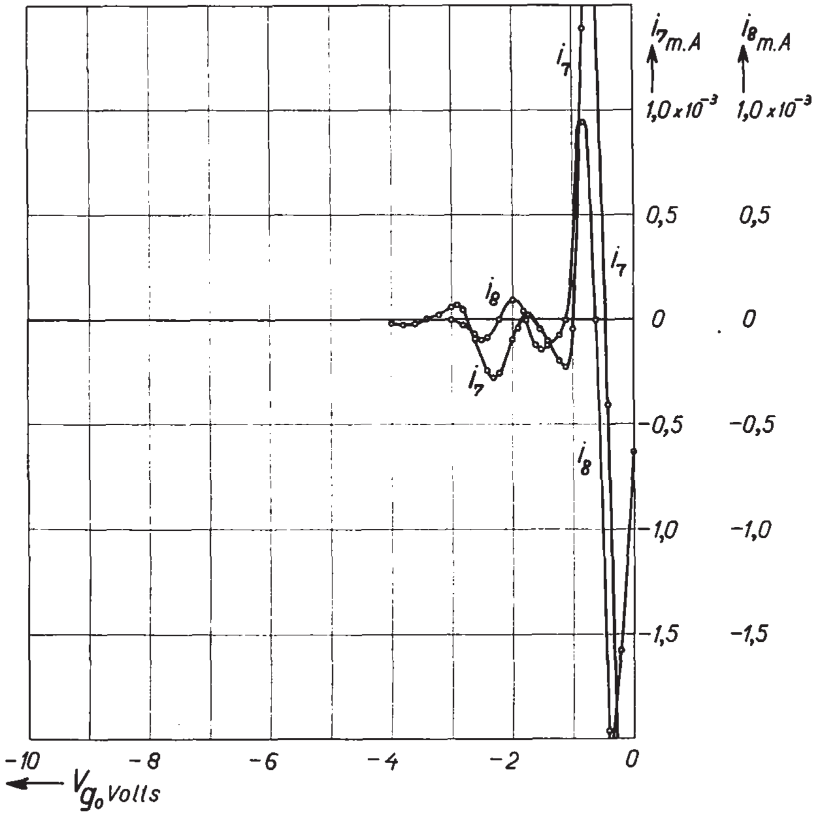

Fig. 5

Let the function under consideration be

$$i_a = f(V_g) \tag{1}$$

and let

$$V_g = V_{g_0} + a \cos \omega t, \tag{2}$$

then

$$i_a = f(V_{g_0} + a \cos \omega t). \tag{3}$$

What we measured is the Fourier analysis of (3), which may be represented by

$$i_a = i_0 + i_1 \cos \omega t + i_2 \cos 2 \omega t + i_3 \cos 3 \omega t + \ldots, \tag{4}$$

where

$$i_n = \frac{1}{\pi} \int_0^{2\pi} f(V_{g_0} + a \cos \omega t) \cos n \omega t \, d \omega t. \qquad (5)$$

If we further *assume our original function* $i_{a_0} = f(V_{g_0})$, *in the domain considered, to be analytic,* so that round any point (i_{a_0}, V_{g_0}) in this domain it can be represented by a Taylor series we may write (3) as:

$$i_a = f(V_{g_0}) + \frac{f'(V_{g_0})}{1!} a \cos \omega t + \frac{f''(V_{g_0})}{2!} (a \cos \omega t)^2 + \frac{f'''(V_{g_0})}{3!} (a \cos \omega t)^3 + \ldots .$$

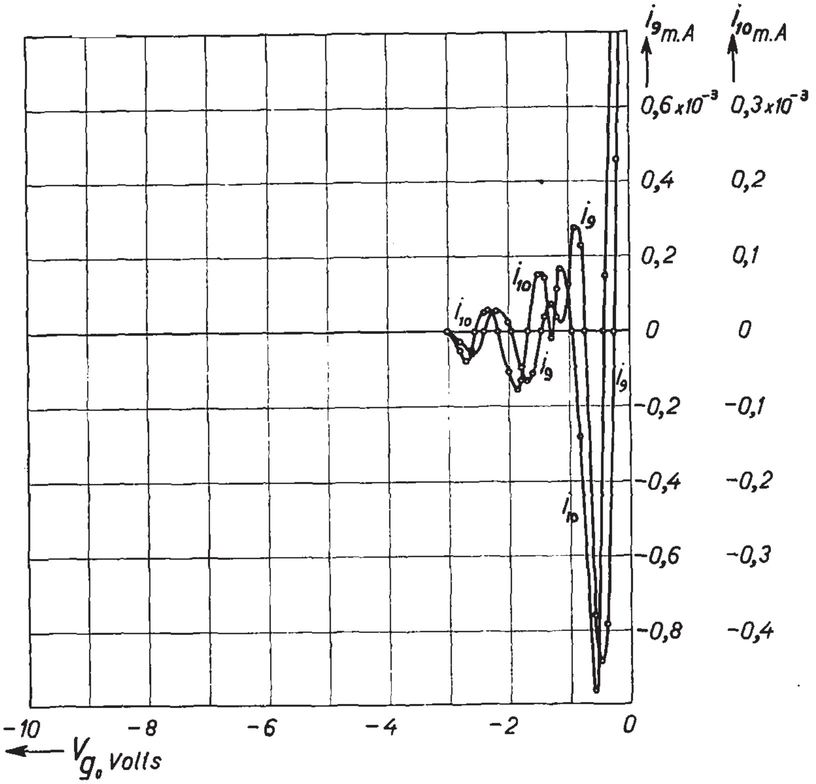

Fig. 6

or

$$i_a = i_{a_0} + \alpha \, a \cos \omega t + \beta (a \cos \omega t)^2 + \gamma (a \cos \omega t)^3 + \ldots , \qquad (6)$$

where

$$\alpha = \frac{1}{1!}\left(\frac{d\,i_a}{d\,V_g}\right)_{V_{g_0}},$$

$$\beta = \frac{1}{2!}\left(\frac{d^2\,i_a}{d\,V_g^{\,2}}\right)_{V_{g_0}},$$

$$\gamma = \frac{1}{3!}\left(\frac{d^3\,i_a}{d\,V_g^{\,3}}\right)_{V_{g_0}},$$

$$\dots\dots\dots\dots\dots$$

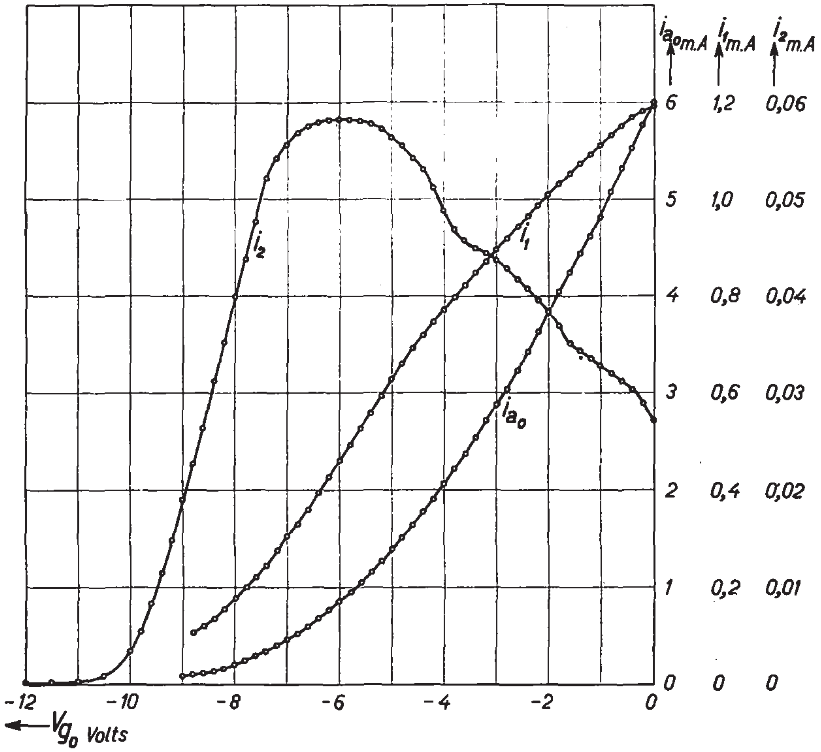

Fig. 7

Any term $(a \cos \omega t)^n$ can be written as

$$\frac{a^n}{2^{n-1}}\left[\cos n\,\omega\,t + \frac{n}{1}\cos\,(n-2)\,\omega\,t + \frac{n(n-1)}{1\,.\,2}\cos\,(n-4)\,\omega\,t + \dots\right];$$

the last term in the finite series in brackets being

$$\frac{n(n-1)\,(n-2)\,\ldots\,\left(\frac{n}{2}+1\right)}{1\,.\,2\,.\,3\,\ldots\,\frac{n}{2}}$$

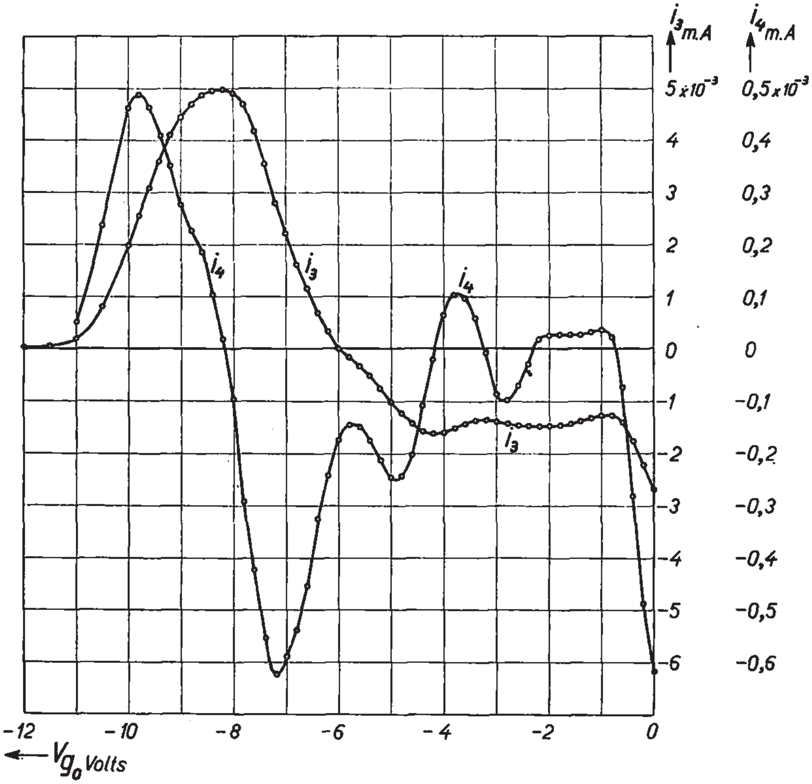

Fig. 8

if n is even, and

$$\frac{n(n-1)\,(n-2)\,\ldots\,\left(\frac{n}{2}+\frac{3}{2}\right)}{1\,.\,2\,.\,3\,\ldots\,\frac{n-1}{2}}\cos\omega t$$

if n is odd.

According to the above-mentioned assumption the terms of (6) may be rearranged, so that we obtain (4), where

$$\left.\begin{aligned}
i_0 &= i_{a_0} + \frac{1}{2}\beta a^2 + \frac{3}{8}\delta a^4 + \frac{5}{16}\zeta a^6 + \frac{35}{128}\theta a^8 + \frac{63}{256}\varkappa a^{10} + \dots\dots,\\
i_1 &= \alpha a + \frac{3}{4}\gamma a^3 + \frac{5}{8}\varepsilon a^5 + \frac{35}{64}\eta a^7 + \frac{63}{128}\iota a^9 + \dots\dots,\\
i_2 &= \frac{1}{2}\beta a^2 + \frac{1}{2}\delta a^4 + \frac{15}{32}\zeta a^6 + \frac{7}{16}\theta a^8 + \frac{105}{256}\varkappa a^{10} + \dots\dots,\\
i_3 &= \frac{1}{4}\gamma a^3 + \frac{5}{16}\varepsilon a^5 + \frac{21}{64}\eta a^7 + \frac{21}{64}\iota a^9 + \dots\dots,\\
i_4 &= \frac{1}{8}\delta a^4 + \frac{3}{16}\zeta a^6 + \frac{7}{32}\theta a^8 + \frac{15}{64}\varkappa a^{10} + \dots\dots,\\
i_5 &= \frac{1}{16}\varepsilon a^5 + \frac{7}{64}\eta a^7 + \frac{9}{64}\iota a^9 + \dots\dots,\\
i_6 &= \frac{1}{32}\zeta a^6 + \frac{1}{16}\theta a^8 + \frac{45}{512}\varkappa a^{10} + \dots\dots,\\
i_7 &= \frac{1}{64}\eta a^7 + \frac{9}{256}\iota a^9 + \dots\dots,\\
i_8 &= \frac{1}{128}\theta a^8 + \frac{5}{256}\varkappa a^{10} + \dots\dots,\\
i_9 &= \frac{1}{256}\iota a^9 + \dots\dots,\\
i_{10} &= \frac{1}{512}\varkappa a^{10} + \dots\dots,\\
&\dots\dots\dots\dots\dots\dots,\\
i_n &= \sum_{p=0}^{\infty} \frac{1}{2^{n+2p-1}} \frac{(n+2p)(n+2p-1)(n+2p-2)\dots\dots(n+p+1)}{p!}\, \nu_{n+2p}\, a^{n+2p},
\end{aligned}\right\} \tag{7}$$

where

$$\nu_1 = \alpha,\ \nu_2 = \beta,\ \nu_3 = \gamma,\ \text{etc.}$$

It therefore follows from (7), that, *when the grid swing a is sufficiently small*, the amplitudes of the different harmonics of the output current i_a, divided by the corresponding power of the amplitude a, and multiplied respectively with proper constants, converges towards the differential coefficients of the valve characteristic, or

$$\frac{d^n i_a}{dV_g^n} = \lim_{a=0} 2^{n-1}\, n!\, \frac{i_n}{a^n}. \tag{8}$$

It was therefore important to investigate experimentally whether on reduction of the grid swing a the above limit was actually found to exist and whether the ratio $i_n/a^n = b_n$ converges towards a constant.

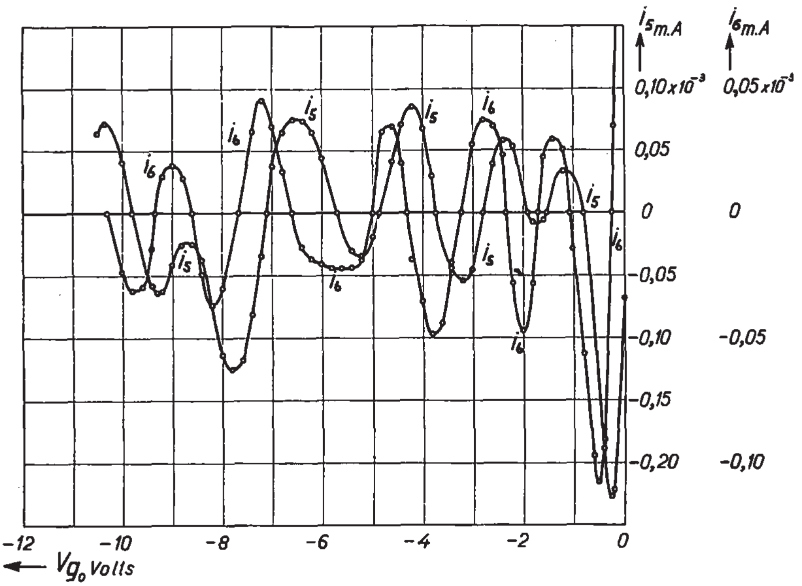

Fig. 9

In the majority of the cases this was indeed found to occur. The accuracy of the experiment was then only limited by the small shot effect which is always present. At some points of the characteristic, however, deviations occurred pointing towards discontinuities in the function under investigation. They were due to critical potentials, causing anomalies in the secondary emission of the electrodes. This will form the subject of a further note. Nevertheless it was felt that a good approximation for practical work could be obtained by assuming the characteristic function to be analytic. The fact moreover, that each curve presents a good approximation to the differential of the one preceding it, also points towards this conclusion.

§ 5. We will now give a new series development for the approximate representation of a triode or tetrode characteristic, based on experimental observations similar to those treated above. The function we wish to investigate is

$$i_a = f(V_g). \tag{1}$$

Taking again

$$V_g = V_{g_0} + a \cos \omega t, \tag{2}$$

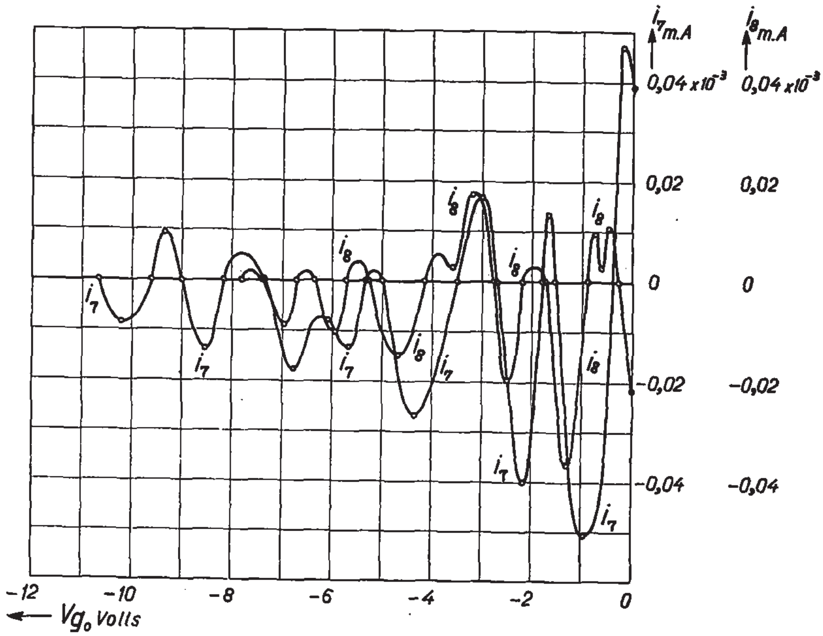

Fig. 10

then the experimental observations give us the values of i_0, i_1 .... in the series

$$i_a = i_0 + i_1 \cos \omega t + i_2 \cos 2 \omega t + \ldots.. \tag{4}$$

Writing further

$$\frac{V_g - V_{g_0}}{a} = \cos \omega t = \mu, \tag{9}$$

then (4) can be written

$$i_a = i_0\, T_0(\mu) + i_1\, T_1(\mu) + i_2\, T_2(\mu) + i_3\, T_3(\mu) + \ldots., \tag{10}$$

where the $T_n(\mu)$'s are the Tchebycheff polynomials of order n and argument μ and the i_n's are the amplitudes of the harmonics given the proper sign. A description of the properties of these polynomials has been given previously [1]).

The amplitudes of the higher harmonics, obtained experimentally, in the way described above, are therefore the coefficients of the development of the characteristic in a series of Tchebycheff *polynomials over the range swept out by the oscillation,* i.e.

$$V_{g_0} + a > V_g > V_{g_0} - a.$$

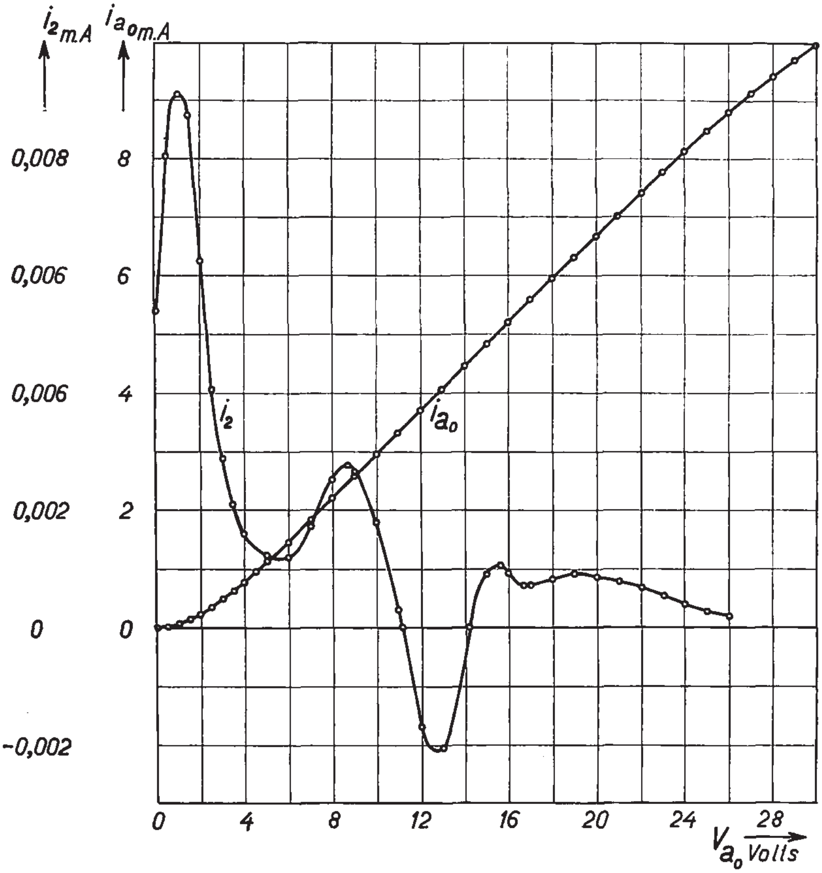

Fig. 11

Obviously the coefficients i_n are functions of the domain considered. Further the orthogonality of the Tchebycheff polynomials has the advantage, that if a higher accuracy is required than that obtained e.g. with n polynomials, the amplitudes of the $(n + 1)$,

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[1]) *Physica*, **1**, 78, 1933.

$(n+2)$th etc. polynomials do *not* affect the amplitudes of the polynomials of the lower orders n, $(n-1)\ldots.\ 0$. If, moreover, for some purpose a power series is required, this can be obtained from the tables of Tchebycheff polynomials published previously[1]) if the numerical values of i_0, i_1 .... of (10) are known.

As an instance we give the following development of the characteristic of the triode E 409 over the grid voltage domain — 28 Volt $< V_g < 0$ Volt:

$$i_a = 11{,}25\,T_0(\mu) + 17{,}5\,T_1(\mu) + 7{,}0\,T_2(\mu) + 4{,}55\,T_3(\mu) - \\ - 0{,}062\,T_4(\mu) - 0{,}15\,T_5(\mu),$$

where $\mu = \frac{V_g + 14}{14}$ (i_a being expressed in milliamperes and V_g in Volts).

The present method (10) of analytically presenting a characteristic obviates the difficulty experienced with a representation like (6) near those points, where the limit (8) does not exist or cannot be determined experimentally.

§ 6. In our opinion the microscopic method of investigating the detailed structure of a diode or triode or other multiple structure tube characteristic as described above, may be of value both as regards the physics of those tubes as well as their technical applications. For on the one hand, it gives a clue to a detailed analysis of secondary emission. Thus an ordinary diode characteristic, such as $i_{a_0} = f(V_{g_0})$ in fig. 12 does not show any irregularity in the grid voltage domain $8 < V_g < 16$ Volt, while e.g. the approximation to its second derivative (also shown in fig. 12) reveals the presence of an anomaly in this region which we traced to be due to secondary high speed electrons from the anode either returning directly towards the cathode or at least influencing the space charge and therefore the primary electron current from the cathode. This will be the subject of a further communication.

On the other hand the fact that the second derivative in fig. 12 crosses the V_{g_0} axis twice, leads one to expect that the detection properties of this diode are greatly impaired at those points. An actual detection test completely confirmed this expectation. Not only was the quality of the detected signal in the domain mentioned

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1) loc. cit.

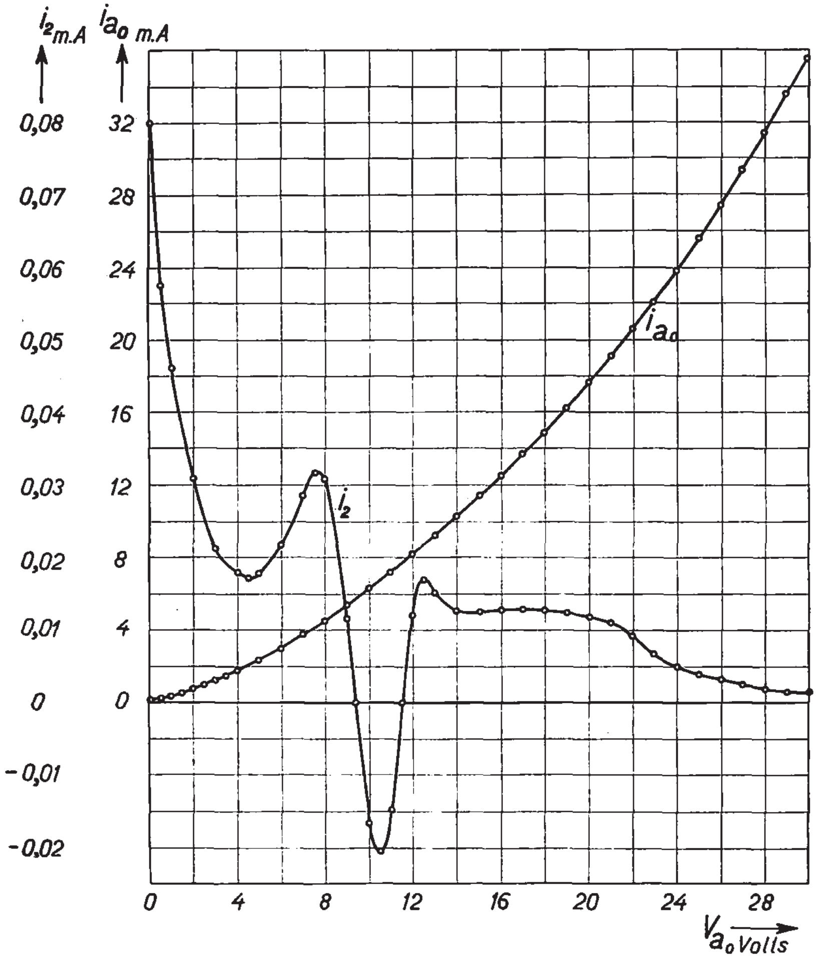

Fig. 12

highly insufficient, but also the mean anode current, as was to be expected from a knowledge of the second differential, showed a drop instead of an increase. Moreover, the detection of a signal of 1 Volt R.M.S. was found to be completely absent in the points $V_{g_0} = 9{,}2$ Volt and $V_{g_0} = 11{,}6$ Volt. For a signal with another amplitude the absence of detection occurs at slightly different values of V_{g_0}.

In conclusion we wish to thank Mr. D. Pasma for his very able assistance in the carrying out of these measurements.

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